Plasticity and Deformation Processes

Plastic deformation and Yielding criteria

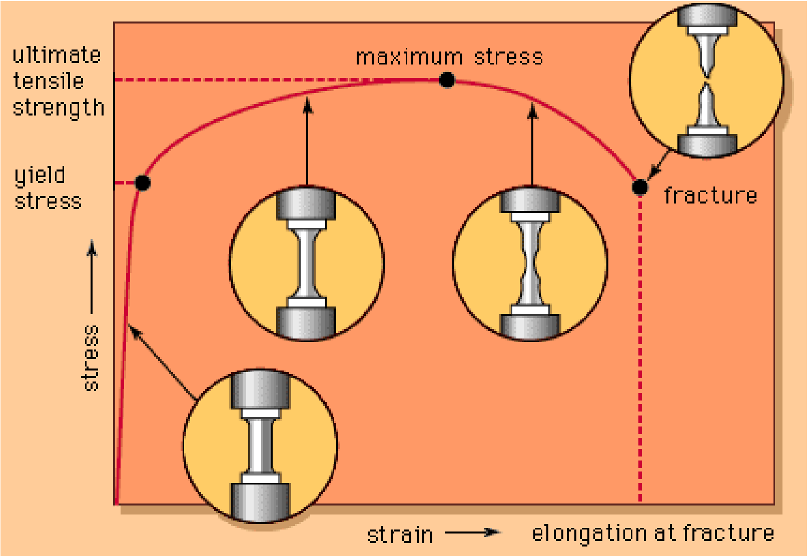
Plasticity

Just as the mechanical properties of a viscoelastic material vary with temperature and strain rate, so do viscoplasticity. Generally plastic deformations are not sensitive to strain rate at low T

Brittle failure is characterized by low strain and rupture that occurs at the highest stress reached.

Ductile failure involves large plastic deformation after yielding, until rupture

Yielded materials exhibit necking and cold drawing. Necking occurs when the cross section is reduced abruptly.
After necking material continues to extend, with the molecules reorienting themselves in the necked region at about constant force. This process of cold drawing produces a material, with its molecules now in a preferred orientation, that is much stiffer.



The stress-strain diagram of a material is obtained by conducting a tensile test on the specimen of material

The initial portion of the stress-strain diagram shows proportionality between the stress applied and the resultant strain according to the Hooke's law

$$\sigma = E\varepsilon$$

The largest value of the stress for which Hooke's law can be used for a material is the **proportional limit**

For ductile materials with a well defined yield point, it coincides with the yield point, for others Hooke's law can be used for stress values slightly larger than the proportional limit (0.2% off-set yield point)

Physical properties of materials like strength, ductility, corrosion resistance may be significantly affected by alloying, heat treatment and manufacturing processes

Although the variation of stress with strain for pure iron and different grades of steel is great and show different yield strength, fracture strength and ductility, they posses the same stiffness

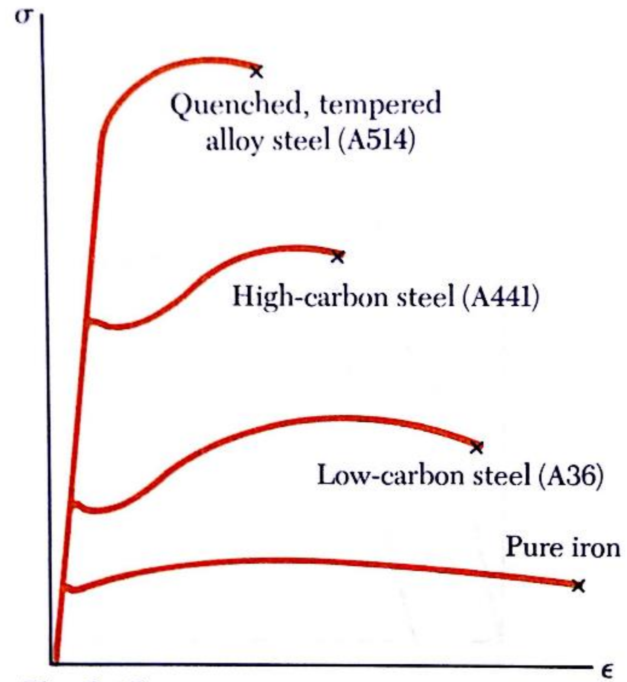
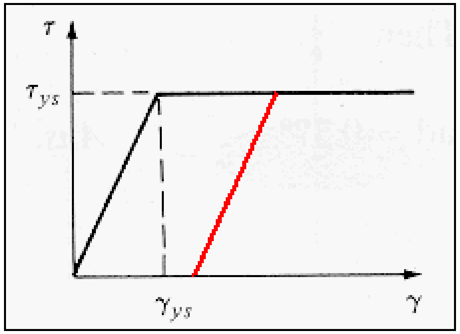


Fig. 2.15

The deformation and failure of plastic materials are hard to characterize because of the non-linear stress-strain relationships and changes in the structure of the material during deformation

The plastic behavior can be simplified by considering an idealized elastoplastic material model which is similar to mild steel



The elastoplastic material strains a small amount elastically at loads below yield stress

When stresses reach the yield strength of the material, it starts yielding and keeps deforming plastically under a constant load

Unloading takes place along a straight line parallel to the elastic region when the load is removed and strain energy is recovered

The segment in the horizontal axis represents the strain corresponding to the permanent deformation.

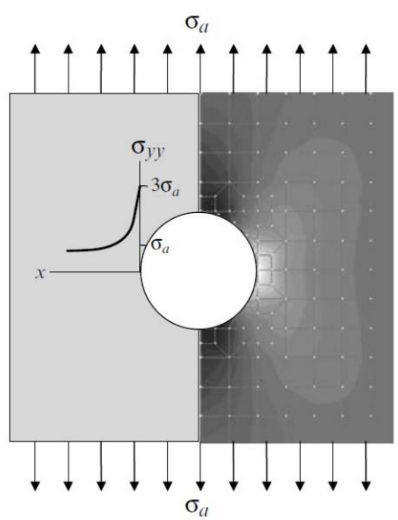
The distribution of stresses on a surface of an elastoplastic material during plastic deformation is as follows:

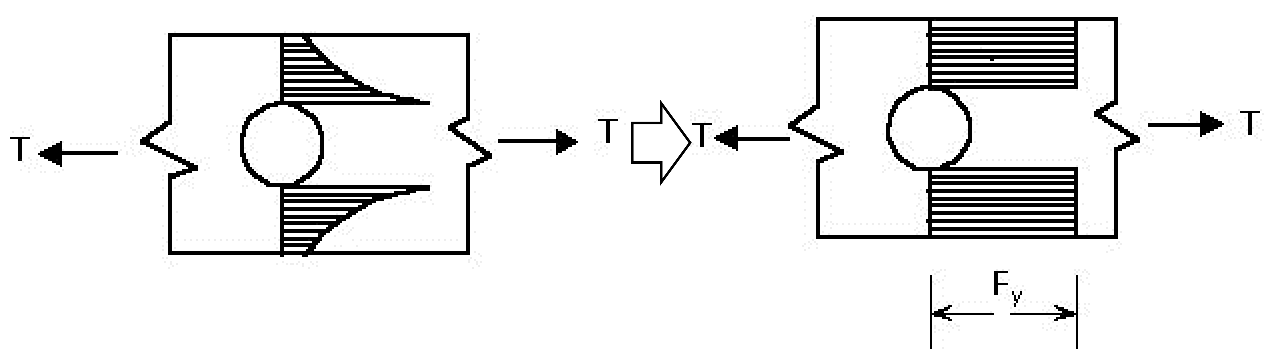
The shaded area under the stress distribution curve represents the load applied. So the area and the value of maximum stress should increase as the load increases.

As the load is increased beyond the yield stress, the stress distribution curve flattens in the vicinity of a void since the stress in the material exceeded the value of yield strength.

The plastic zone where yield takes place keeps expanding as the load is further increased, until it reaches the edges of the plate.

At that point the distribution of stresses across the material is uniform above the yield stress and the load is the largest which may be applied to the bar without causing rupture.





A mathematical theory on the fracture behavior of plastic materials is not present since it usually occurs at high strains, which are mathematically difficult to formulize

Fracture mechanics of brittle materials as described by Griffith serves as a reference to understand the failure of plastics

According to Griffith, a fracture results in the formation of two new surfaces on each side of the crack and that the formation of these surfaces requires energy. This energy is stored as changes in bond length throughout the rest of the material as it is stretched

Fracture requires the transmission of this energy to the fracture surfaces, at the same time relaxing the strain in the area from which energy has been released

Propagation of the crack depends on the crack length
The critical length should be exceeded for the surrounding material to transmit energy to cause brittle fracture.

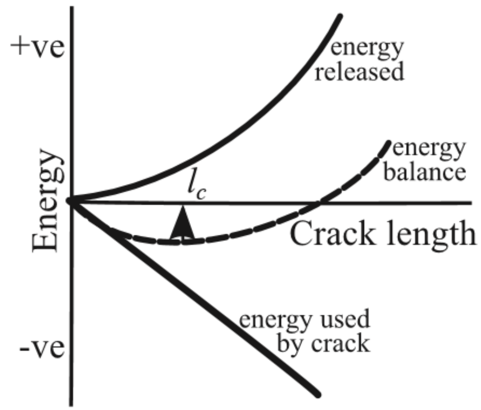


Figure 1.21. The energy conditions associated with the propagation of cracks showing the derivation of the critical crack length, *lc*. The term "+ve" indicates strain energy released by the crack; "–ve" indicates energy used to make new surfaces as the crack progresses.

Based on Griffith's theory, two general criteria need to be satisfied for a piece of material to fracture:

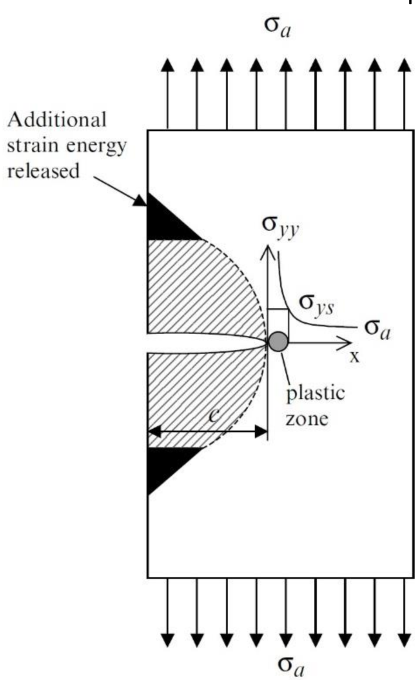
- The bonds at the crack tip must be stressed to the point of failure

(the critical stress intensity, K_C must be reached)

- There must be sufficient elastic strain energy (work of fracture) available at the tip of the crack to propagate it to the surface

In engineering, the critical stress intensity K_{Ic} is known as the fracture toughness

Toughness of a material is important to resist the initiation and propagation of a crack.



Stress field in the vicinity of a sharp crack tip as a function of r and θ:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$ Irwin equation

where $K_I = \sigma_a Y\sqrt{\pi c}$

determines the magnitude of stress at the crack tip

The subscript I in K_I is associated with tensile loading

Stress intensity factors exist for other types of loading

These stress intensity factors are additive

The factor for a complicated system of loads can be derived from the addition of the factors determined for each load separately

Type I loading is the most common type that leads to brittle failure

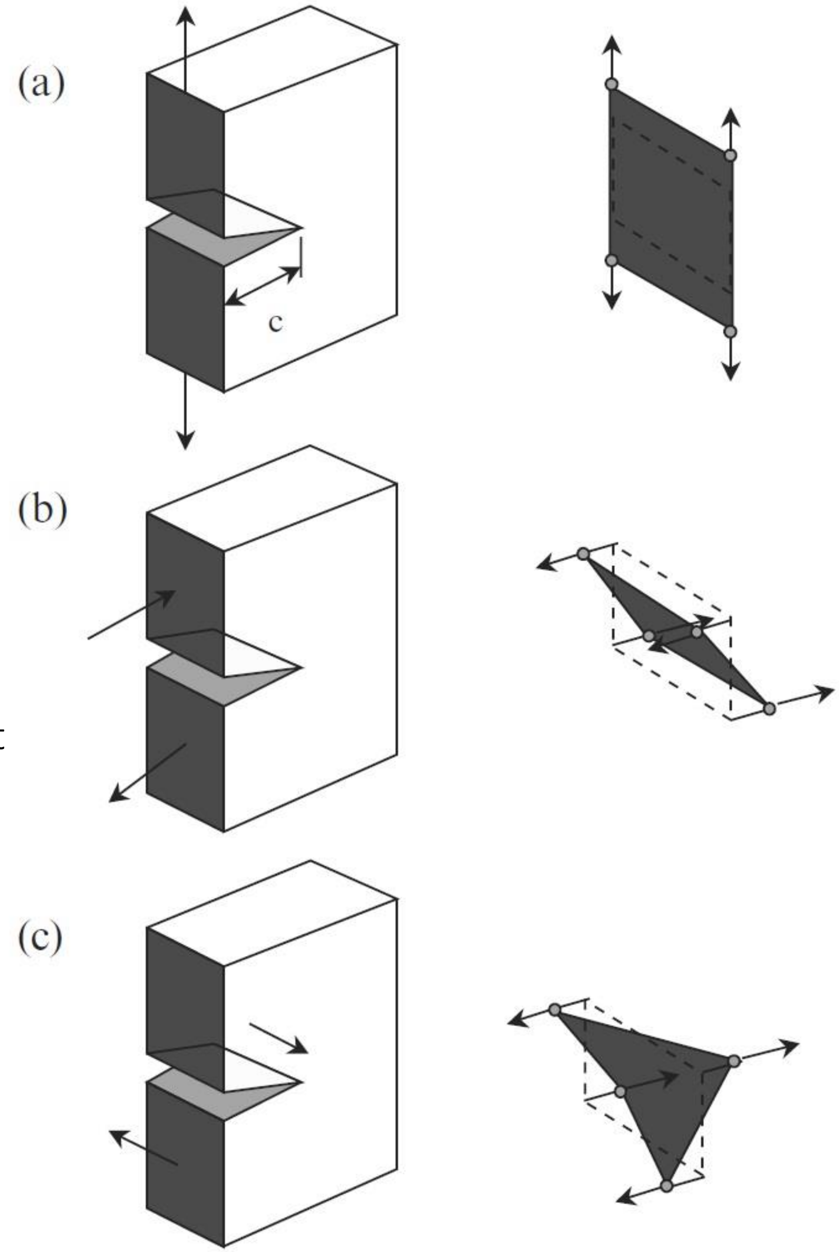


Fig. 2.4.2 Three modes of fracture. (a) Mode I, (b) Mode II, and (c) Mode III. Type I is the most common. The figures on the right indicate displacements of atoms on a plane normal to the crack near the crack tip.

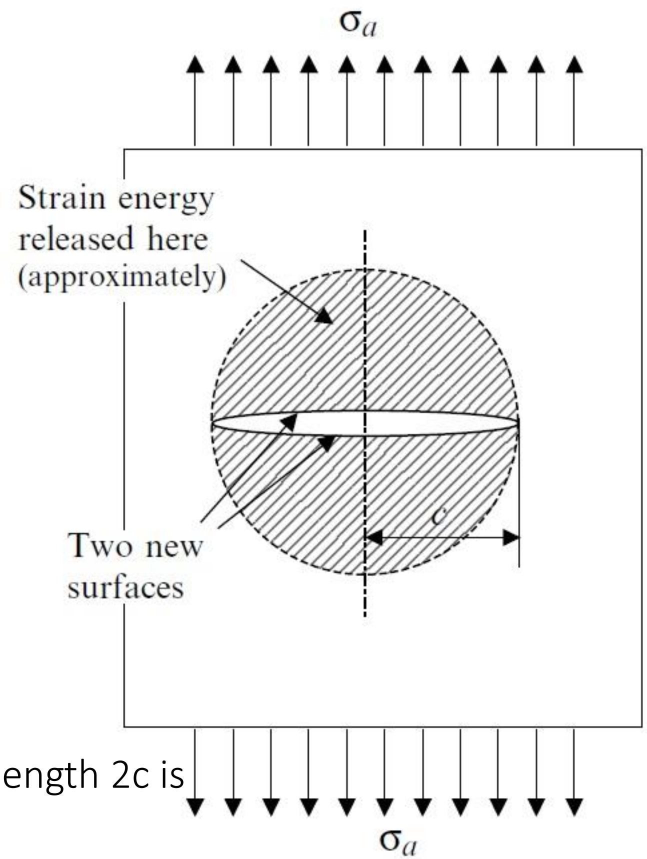
The second condition:
For an increment of crack extension, the amount of strain energy released must be greater than or equal to that required for the surface energy of the two new crack faces

or $$\frac{dU_s}{dc} \geq \frac{dU_\gamma}{dc}$$ Griffith equation

where U_s is the strain energy, U_γ is the surface energy, and dc is the crack length increment

Griffith showed that show the strain energy released by introducing a double-ended crack of length 2c in an infinite plate of 1 m width under a uniformly applied stress σ_a is

$$U_s = \frac{\pi \sigma_a^2 c^2}{E} \text{ Joules}$$



The total surface energy for two surfaces of 1 m width and length 2c is

$$U_\gamma = 4\gamma c \text{ Joules}$$

where γ is the fracture surface energy of the solid. It may additionally involve energy dissipative mechanisms such as microcracking, phase transformations, and plastic deformation

So taking the derivative of strain and fracture surface energies with respect to c gives the strain energy release rate and the surface energy creation rate (J/m)

$$\frac{\pi \sigma_a^2 c}{E} \geq 2\gamma$$

The strain energy release rate per increment of crack length is a linear function of crack length
The required rate of fracture surface energy per increment of crack length is a constant

The relationships between surface energy, strain energy, and crack length:

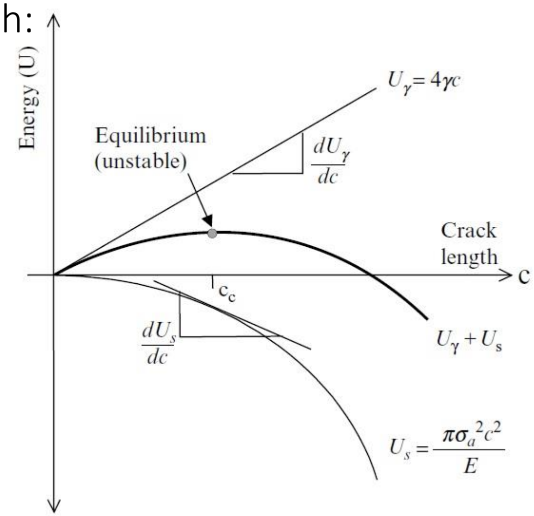
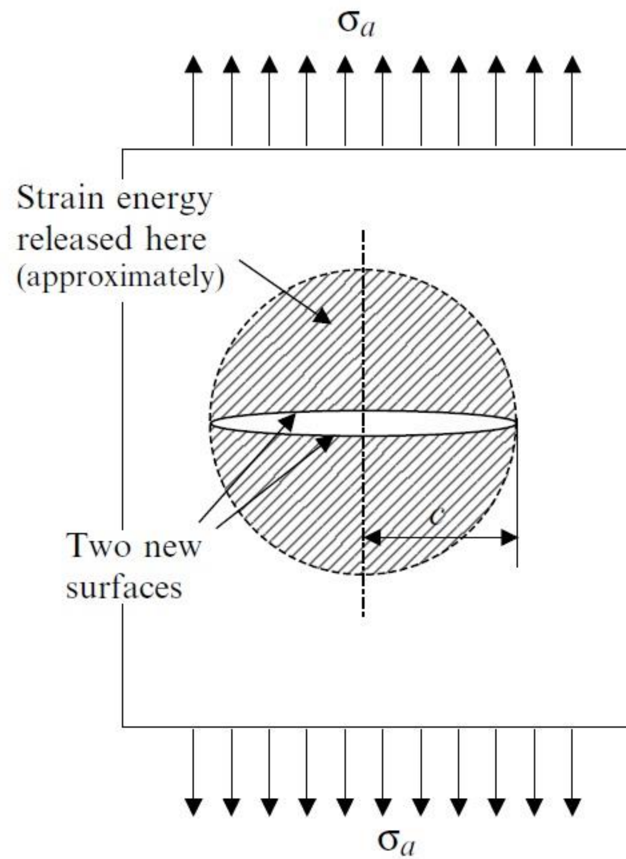


Fig. 2.3.2 Energy versus crack length showing strain energy released and surface energy required as crack length increases for a uniformly applied stress as shown in Fig. 2.3.1. Cracks with length below c_c will not extend spontaneously. Maximum in the total crack energy denotes an unstable equilibrium condition.

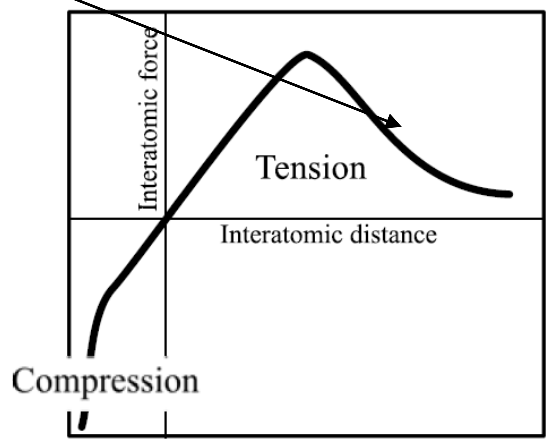


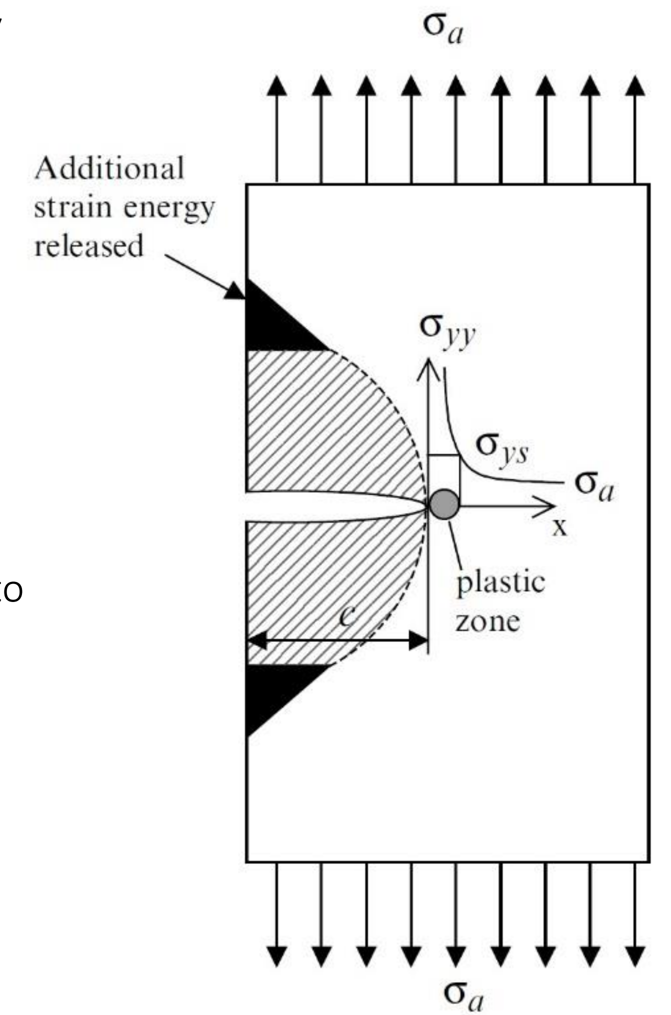
$$c_c = \frac{2\gamma E}{\pi \sigma_a^2} \Rightarrow K_{Ic} = \sigma_a Y \sqrt{\pi c_c}$$

The stress intensity (not K_I) at the crack tip (r=0) is infinity according to the Irwin equation

In practice, the stress at the crack tip is limited to at least the yield strength of the material

This is the result of the plastic zone within a certain distance of the crack tip for metals
For ceramics, nonlinear elastic deformation consumes energy





The elastic material outside the plastic zone transmits stress to the material inside the zone

The plastic zone is usually small (σ_{yy} is proportional to $1/r^{1/2}$)

So the strain energy release rate is not affected much by the energy dissipation mechanisms within the zone

Energy in a plastic material can be used up or dissipated before it can reach the crack tip to contribute to the new fracture surface

Rubber fibers and particles are added to various polymers for additional energy storage mechanism and the resultant toughening effect

Toughness of metals results from dissipation of far more energy than is needed for propagation of the crack by slippage of crystal planes or by dislocation motion

Ductility results from the plastic flow of material as the dislocation move at applied stresses below the ultimate strength

Irwin modified Griffith's equation to take into account the non-reversible energy absorption mechanisms within the plastic zone:

$$\frac{dU_s}{dc} \geq \frac{dU_\gamma}{dc} + \frac{dU_p}{dc}$$

The right hand side of the equation is called the crack resistance which indicates the minimum energy required for crack extension
This minimum energy is called the work of fracture which is a measure of toughness

Damping is the measure of viscous dissipation of mechanical energy in the microstructure by various mechanisms consisting of stress-induced movement of defects

Damping or Internal friction is defined as the capacity of a material to convert its mechanical energy of vibration into heat that is dissipated in the material (same as tan δ)

Point defects give rise to damping in the range of low to intermediate levels
Line defects (dislocations) give rise to damping levels in the intermediate to high range
Planar defects (boundaries of various types) give rise to damping levels in the high range

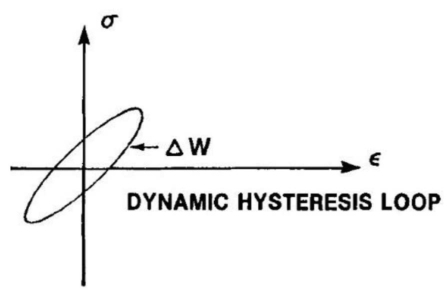
They generally operate in two major mechanisms:

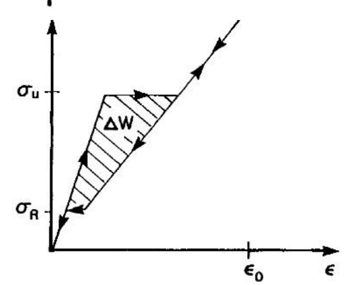
- Dynamic hysteresis

produced by the stress-aided ordering of defects overcoming local barriers
For example, diffusion-controlled rearrangement of solute atoms

- Static hysteresis

Produced by an "unpinning" process, or "breakaway“ process, at the defect level
For example in the case of metals, the mechanical unpinning of dislocations
After a linear elastic response to the stress, a breakaway strain is produced at a critical value of the stress for stress-induced movement of pinned dislocations.
As the unpinned dislocations collapse elastically during unloading, they become repinned at a much lower stress





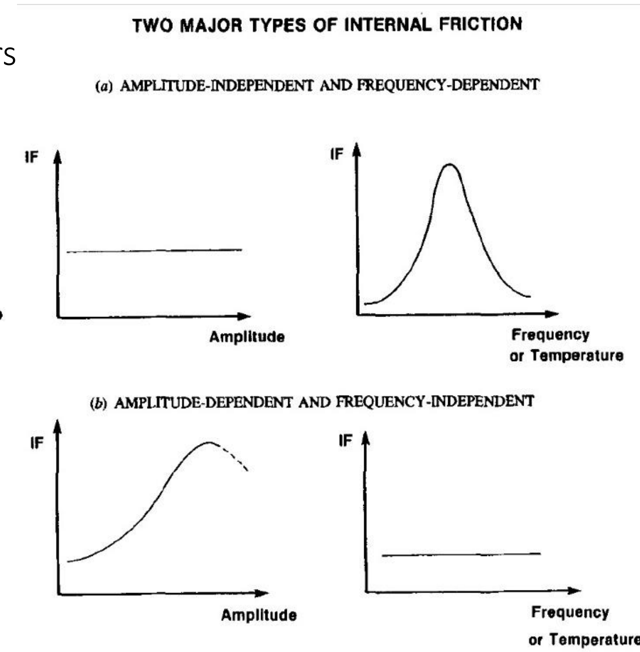
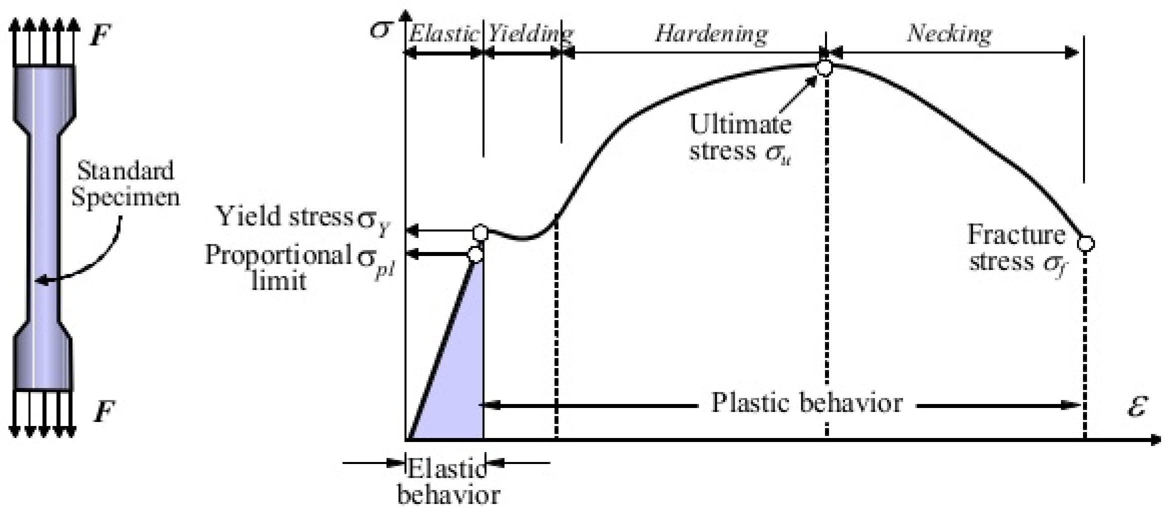


Fig. 8—Schematic diagrams of internal friction as a function of strain amplitude and temperature for (*a*) amplitude-independent (dynamic hysteresis) and (*b*) amplitude-dependent (static hysteresis) damping.

Materials behave elastically until the deforming force increases beyond the yield stress. At that point, the material is irreversibly and permanently deformed.

Irreversible deformation at normal temperatures cause the dislocations to accumulate, interact with one another, and serve as pinning points or obstacles that significantly impede their motion. This leads to an increase in the yield strength of the material and a subsequent decrease in ductility.

Its extent is dependent on the material and the dislocation density



Because dislocation motion is hindered, plastic deformation cannot occur at normal stresses. The yield stress increases as a result.

At a stress lower than the yield stress, a cold-worked material will continue to deform using the only mechanism available: elastic deformation and the modulus of elasticity is unchanged. With increasing stress the strain-field interactions are overcome and plastic deformation resumes. It has now become a brittle material. If dislocation motion and plastic deformation have been hindered enough by dislocation accumulation, and stretching of electronic bonds and elastic deformation have reached their limit, a third mode of deformation occurs: fracture.

Increase in the number of dislocations is a quantification of work hardening. Plastic deformation occurs as a consequence of work being done on and energy added to a material. In addition, the energy is almost always applied fast enough and in large enough magnitude to not only move existing dislocations, but also to produce a great number of new dislocations.

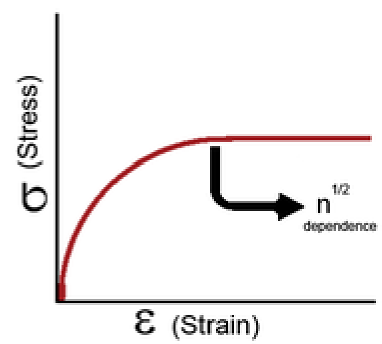
$$\Delta\tau \ = Gb\rho^{1/2}$$

Work hardening has a half root dependency on the number of dislocations. The material exhibits high strength if there are either high levels of dislocations (greater than 10^{14} dislocations per m^2) or no dislocations. A moderate number of dislocations (between 10^7 and 10^9 dislocations per m^2) typically results in low strength

Work hardening phenomenon is formulated as a power law relationship between the stress and the amount of plastic strain:

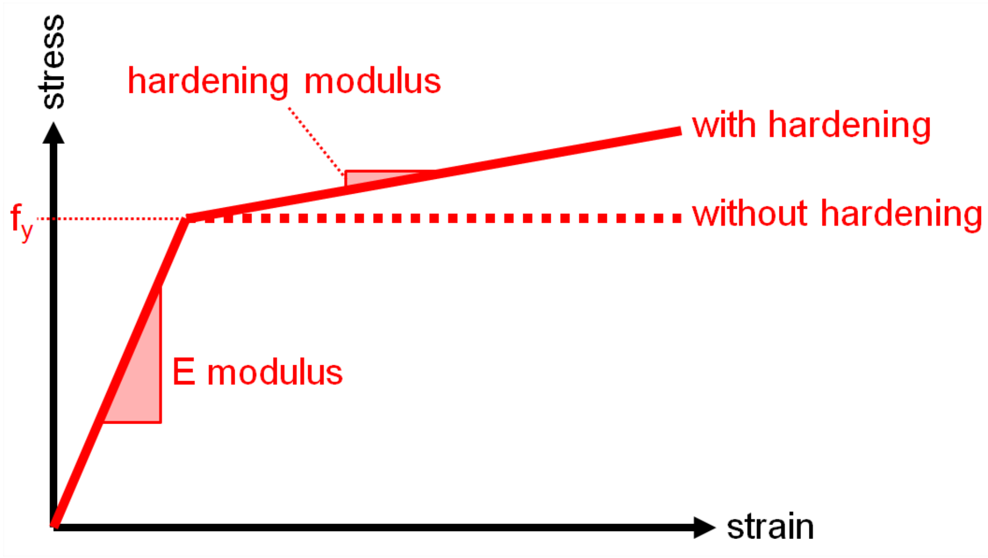
$$\sigma = K\epsilon_p^n \quad \text{or} \quad \sigma = \sigma_y + K\epsilon_p^n$$

where σ is the stress, σ_y is the yield stress, K is the strength index or strength coefficient, ε_p is the plastic strain and n is the strain hardening exponent.



Elastoplastic material is a model for easily plastically deformed materials (e.g. at high T) that simplifies the calculations significantly

In practice all materials harden to an extent and mostly non-linearly without a definite hardening modulus



Calculation of the plastic deformations similarly to calculation of elastic deformations using Hooke's law is only possible by using a dynamic modulus which is a definite function of applied strain

Deformation theory helps us do that once we determine the yielding condition and the effective stress state

The tangent modulus K is the slope of the stress-strain curve in the plastic region and in general it changes during a deformation

At any instant of strain, the increment in stress $d\sigma$ is related to the increment in true strain $d\varepsilon$ by

$$d\sigma = K d\varepsilon$$

The strain increment after yield consists of both elastic and plastic strains

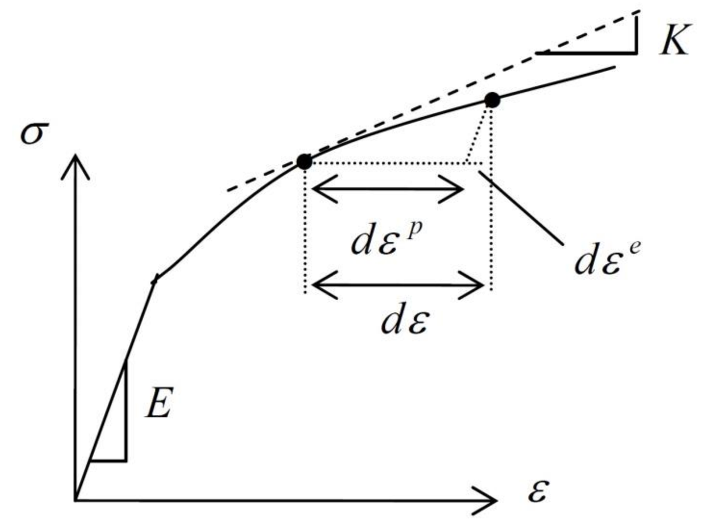
$$d\varepsilon = d\varepsilon^e + d\varepsilon^p$$

The stress and plastic strain increments are related by the plastic modulus H:

$$d\sigma = H d\varepsilon^p$$

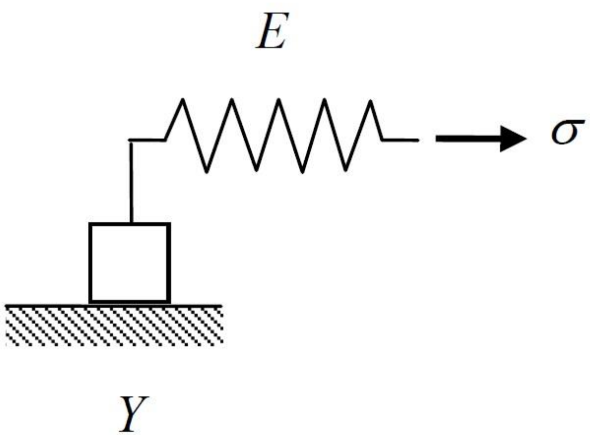
Such that

$$\frac{1}{K} = \frac{1}{E} + \frac{1}{H}$$

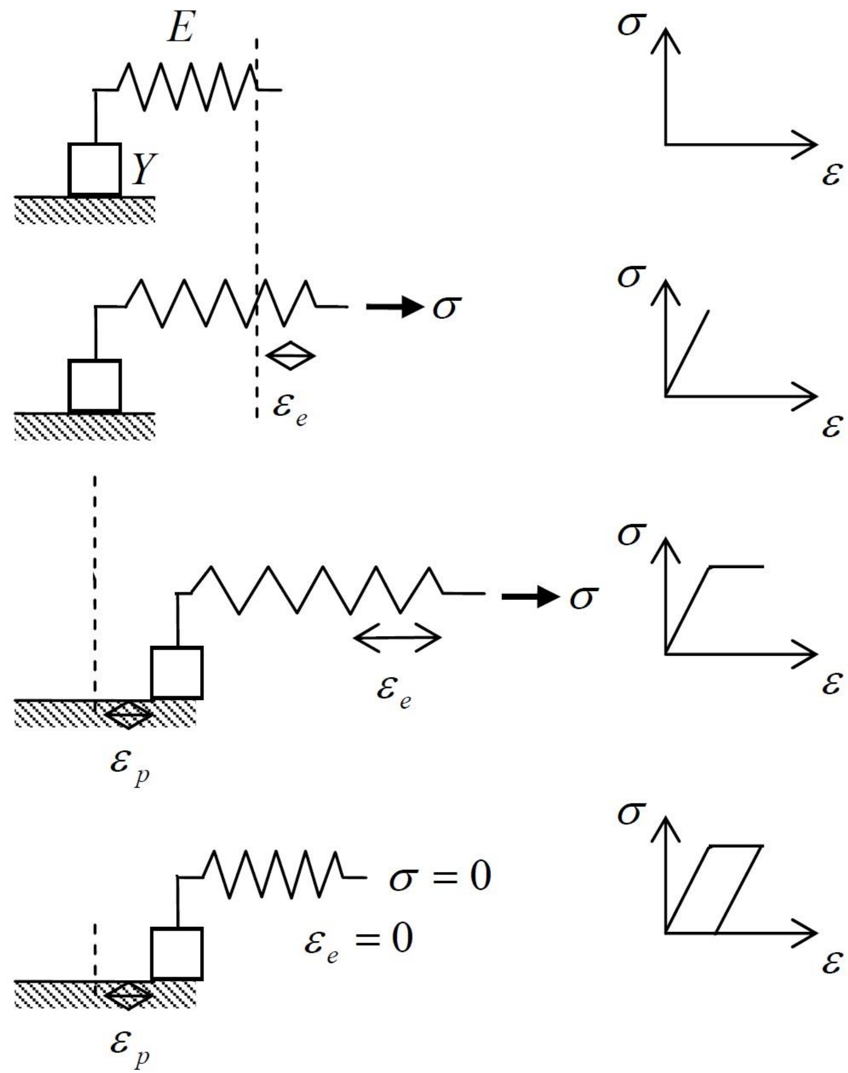


Just like we model viscoelastic behavior with combinations of perfectly elastic springs and perfectly viscous dashpots, we can model the behavior of plastic materials using friction blocks in place of dashpots

The elastoplastic model incorporates a friction block with a yield stress Y, connected in series with a free spring with modulus E



The spring extends elastically until a stress of Y is applied. Then there is only the movement of the friction block and plastic deformation. The stress cannot exceed the yield stress. If unloaded the block stops moving, the spring contracts and the stress returns to zero, leaving a permanent strain



The elastoplastic model with linear strain hardening is a more realistic model for metals

It incorporates a second spring with stiffness H, parallel with the friction block and representing strain hardening

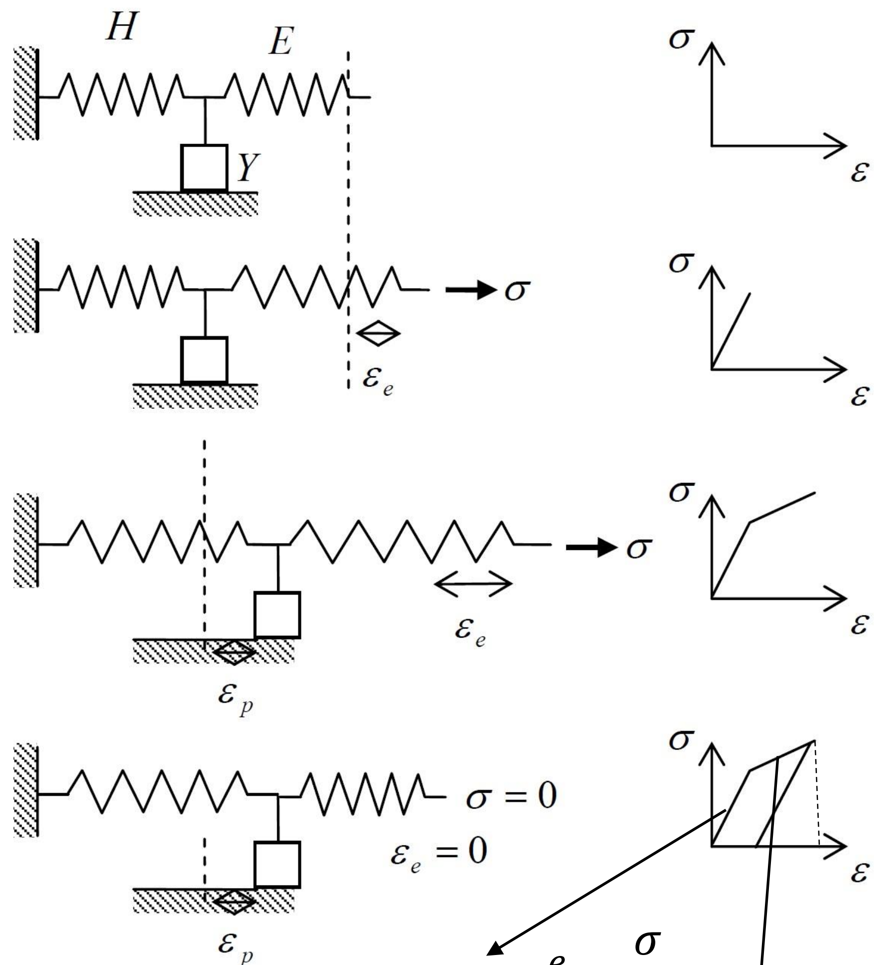
An increasing stress needs to applied after the yield stress Y is reached in order to keep the block moving

Elastic strain continues to occur due to further elongation of the free spring

The stress is then consumed by the plastic deformation by the moving block and the remaining stress *σ-Y* is carried by the hardening spring

Upon unloading the block locks, the stress in the hardening spring remains constant while the free spring contracts

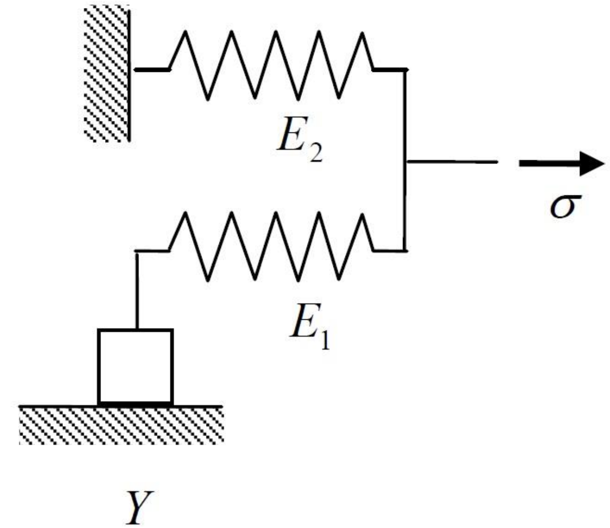
At zero stress there is negative stress at the block as a response to the strain in the spring



$$\varepsilon = \varepsilon^e = \frac{\sigma}{E}$$

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + \frac{\sigma - Y}{H}$$

Example – Consider the plasticity model shown

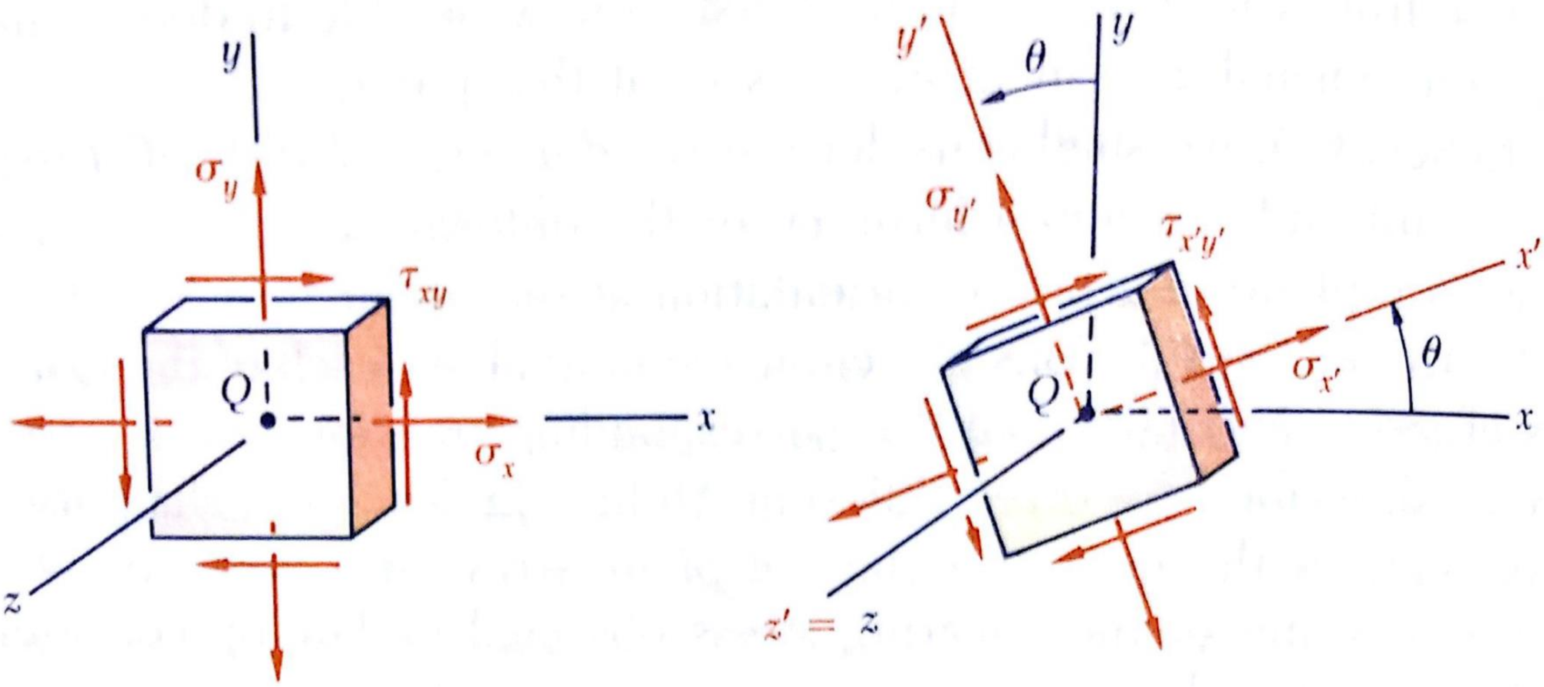


Draw the stress-strain diagram
What are the elastic modulus, yield stress, tangent and plastic moduli?

Yielding depends on the magnitude of the normal and shear stresses applied to a material and also on the local stresses generated at some plane (slip-plane) within the material

Consider a planar material stressed in two directions

A state of plane stress exists at a point Q with $\sigma_z = \tau_{zx} = \tau_{zy} = 0$. The state of plane stress is defined by the stress components $\sigma_x, \sigma_y, \tau_{xy}$ associated with the material shown:



If the material is rotated through an angle θ about the z axis, the stress components change to $\sigma_x{}', \sigma_y{}', \tau_{xy}{}'$ which can be expressed in terms of $\sigma_x, \sigma_y, \tau_{xy}$ and θ

Consider a prismatic element with faces respectively perpendicular to the x, y and x’ axes:

If the area of the oblique face is ΔA, the areas of the vertical and horizontal faces are equal to ΔA cosθ, and ΔA sinθ respectively.

The mechanical equilibrium along the x’ and y’ axes require that

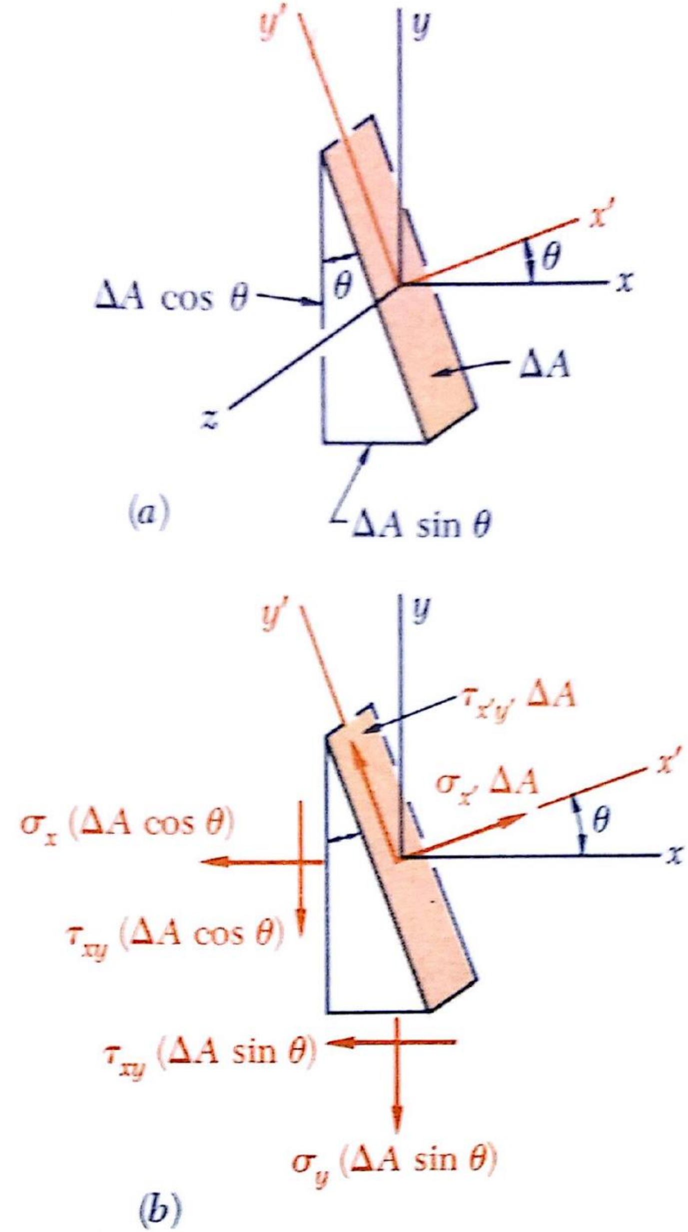
$$\sum F_{x'} = 0,\ \sigma_x{}'\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta - \sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

$$\sum F_{y'} = 0,\ \tau_{x'y'}\Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta - \sigma_y(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta = 0$$

The first equation is solved for $\sigma_x{}'$ and the second for $\tau_{x'y'}$ as

$$\sigma_x{}' = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$



After simplifications using trigonometric substitutions we obtain the normal and shear stresses on the rotated material as

$$\sigma_x{}' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_y{}' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

The expression for the normal stress $\sigma_y{}'$ is obtained by replacing θ by the angle θ+90 that the y' axis forms with the x axis.

Adding the two normal stresses we see that

$$\sigma_x{}' + \sigma_y{}' = \sigma_x + \sigma_y$$

In the case of plane stress, the sum of the normal stresses exerted on a cubic material is independent of the orientation of the material since $\sigma_z = \sigma_{z'} = 0$

The equations obtained for the normal and shear stresses in the rotated material under plane stress condition are the parametric equations of a circle

If we plot a point M in the rectangular axes with the coordinates $(\sigma_x{}', \tau_{x'y'})$ for any given value of the parameter θ, all the other possible points will lie on a circle.

$$\sigma_x{}' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

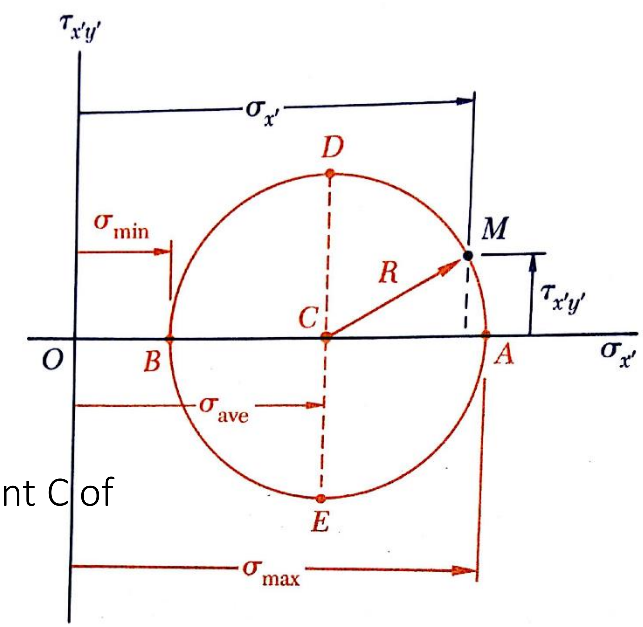
The angle θ in the equations can be eliminated by algebraic simplifications and addition of the two equations:

$$\left(\sigma_x{}' - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}{}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2$$

Where $\frac{\sigma_x+\sigma_y}{2} = \sigma_{ave}$ and $\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2 = R^2$

So $(\sigma_x{}' - \sigma_{ave})^2 + \tau_{x'y'}{}^2 = R^2$

Which is the equation of a circle of radius R centered at the point C of coordinates $(\sigma_{ave}, 0)$



The point A where the circle intersects the horizontal axis is the maximum value of the normal stress $\sigma_x{}'$ and the other intersection point B is the minimum value. Both points correspond to a zero value of shear stress $\tau_{x'y'}$.
These are the **principle stresses**.

Since $\sigma_{max} = \sigma_{ave} + R$ $\sigma_{min} = \sigma_{ave} - R$

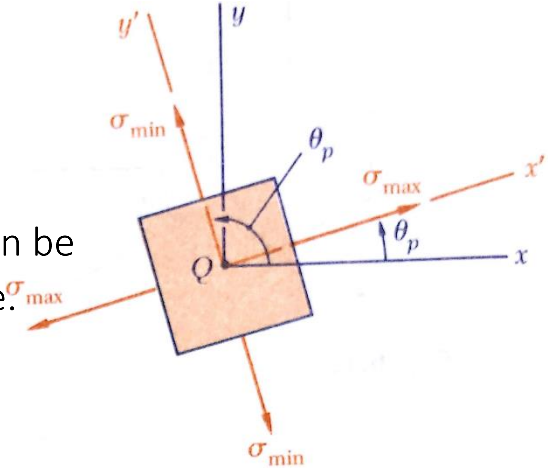
$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

The rotation angles that produce the principal stresses with no shear stress is obtained from the equation of shear stress

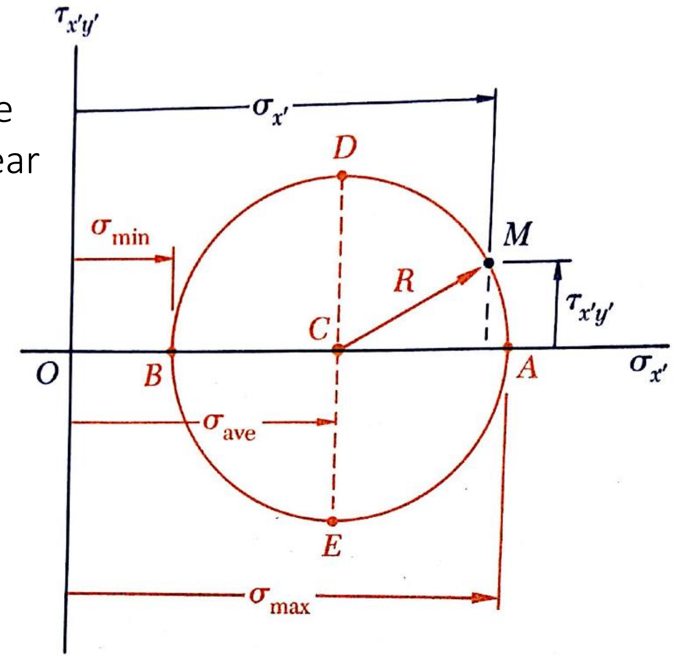
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = 0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

This equation gives two θ_p values that are 90 apart. Either of them can be used to determine the orientation of the corresponding rotated plane. These planes are the principal planes of stress at point Q



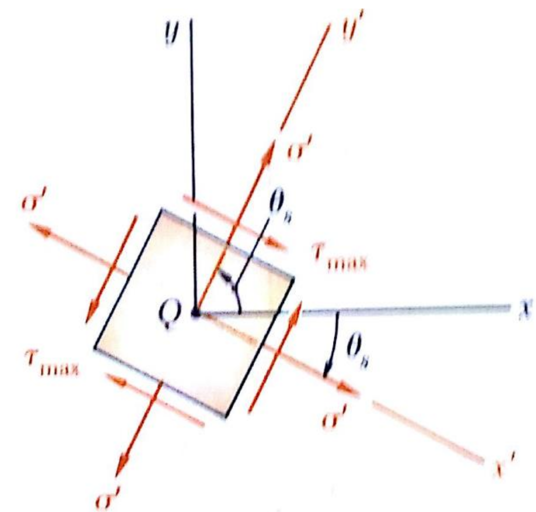
The points D and E are located on the vertical diameter of the circle corresponding to the largest numerical value of the shear stress $\tau_{x'y'}$. These points have the same normal stresses of σ_{ave}. So the rotation that produces the maximum shear stresses can be obtained from the normal stress equations.



$$\sigma_x' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

This equation gives two θ_s values that are 90 apart. Either of them can be used to determine the orientation of corresponding rotated plane that produces the maximum shear stress which is equal to



$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

The normal stress corresponding to the condition of maximum shear stress is

$$\sigma_x{}' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

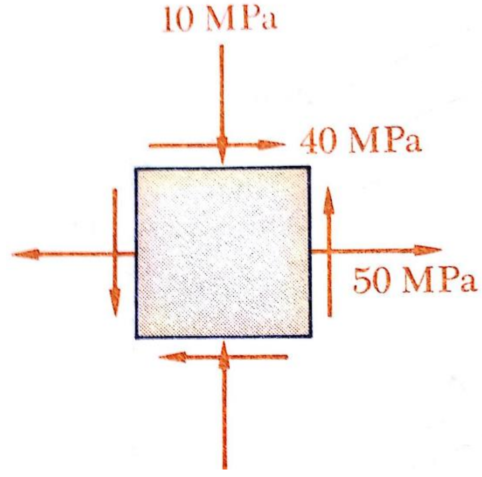
Also

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\left(\tan 2\theta_p\right)^{-1} = -\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)^{-1}$$

This means that the angles θ_s and θ_p are 45 apart

So the planes of maximum shear stress are oriented at 45 to the principal planes

Example – Determine the principal planes, principle stresses, maximum shear stress and the corresponding normal stress for the state of plane stress shown



Yield criteria for ductile materials under plane stress

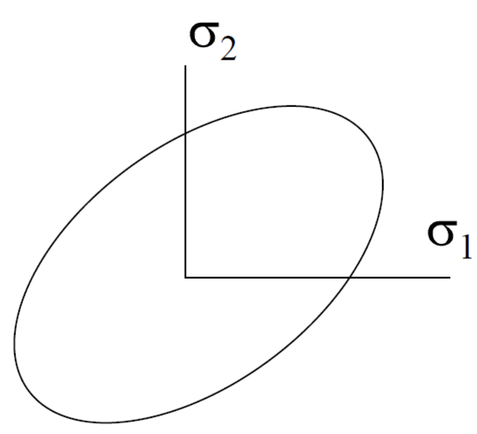
When a ductile material is under uniaxial stress, the value of the normal stress σ_x which will cause the material to yield can be determined simply from a stress-strain diagram obtained by a tensile test.

The material will deform plastically when $\sigma_x > \sigma_{Yield}$

On the other hand when a material is in a state of multiaxial stress, the material will yield when the maximum value of the shear stress exceeds the corresponding value of the shear stress in a tensile-test specimen as it starts to yield.

Maximum shear stress criterion is based on the observation that yield in ductile materials is caused by slippage of the material along oblique surfaces and is due primarily to shear stresses.

In the plane stress condition the material can be represented as a point under principal stresses σ_a, σ_b



Recall that the maximum value of shear stress at a point under a centric axial load is equal to half the value of the corresponding normal axial stress.

Thus at yielding

$$\tau_{max} = \frac{1}{2}\sigma_Y$$

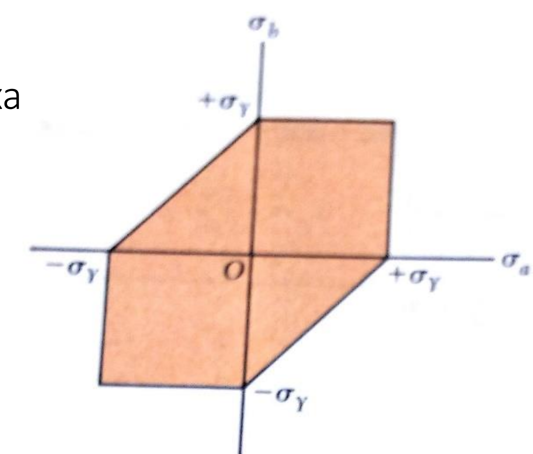
Also for plane stress condition if the principle stresses are both positive or both negative, the maximum value of the shear stress is equal to $\frac{1}{2}|\sigma_{max}|$

Therefore $\quad |\sigma_a| > \sigma_Y$ or $|\sigma_b| > \sigma_Y$

If the maximum stress is positive and the minimum stress negative, the maximum value of the shear stress is equal to $\frac{1}{2}(|\sigma_{max}| - |\sigma_{min}|)$

Therefore $\quad (|\sigma_a| - |\sigma_b|) > \sigma_Y$

These relations produce a hexagon in the xy plane, called Tresca's hexa will be represented in the figure by a point.



Maximum distortion energy criterion is based on the determination of the distortion energy in a given material, which is the energy consumed by a change in the shape of the material.

Also called von Mises criterion, it states that a material will yield when the maximum value of the distortion energy per unit volume exceeds the distortion energy per unit volume required to cause yield in a tensile test specimen.

The distortion energy in an isotropic material under plane stress is

$$U_d = \frac{1}{6G}({\sigma_a}^2 - \sigma_a\sigma_b + {\sigma_b}^2)$$

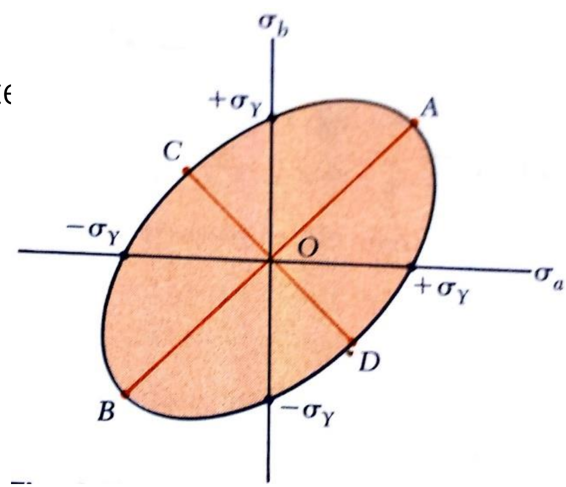
In the case of a tensile test specimen yielding at σ_Y

$$U_Y = \frac{1}{6G}({\sigma_Y}^2)$$

Thus the maximum distortion energy criterion indicates that the mate

$${\sigma_a}^2 - \sigma_a\sigma_b + {\sigma_b}^2 > {\sigma_Y}^2$$

This equation produces an ellipse in the principal stress plane



The von Mises yield criterion is given by

$$\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = 2\sigma_y$$

Or

$$\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left({\tau_{yz}}^2 + {\tau_{zx}}^2 + {\tau_{xy}}^2\right)} = 2\sigma_y$$

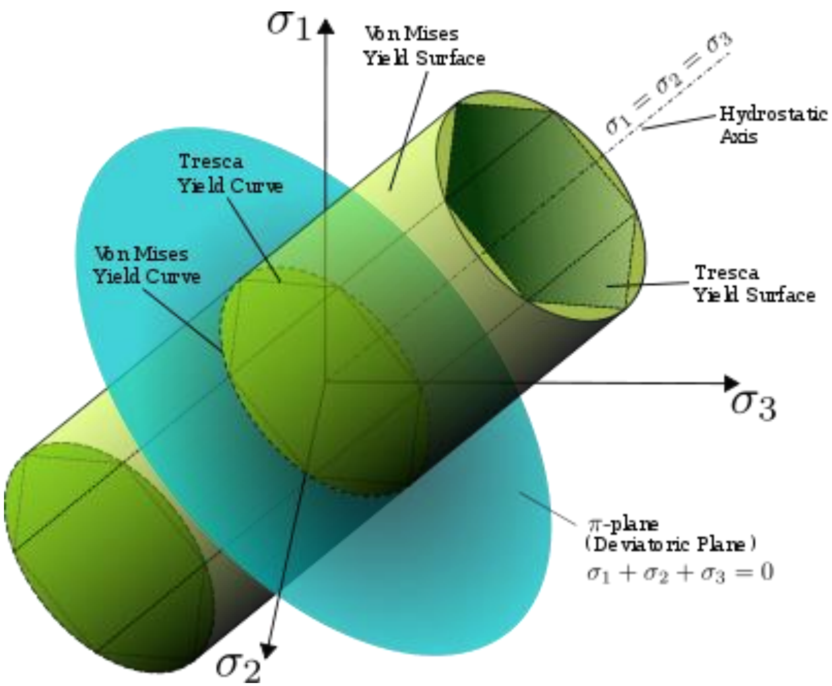
In terms of effective stress the criterion is

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_y$$

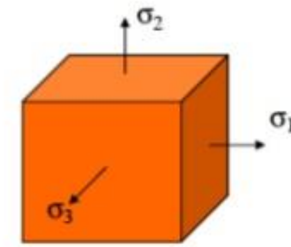
$$\sigma_{eff} = \frac{\sqrt{2}}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left({\tau_{yz}}^2 + {\tau_{zx}}^2 + {\tau_{xy}}^2\right)} = \sigma_y$$

For plane states of stress, the yield condition is the interaction of the cylinder with the principal stress plane, which is a yield ellipse

The von Mises yield criterion is visualized as a circular cylinder in the stress space



✓ **Body subjected to principal stresses :**



$$U = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3)$$

$$U=1/2E\,[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\,\nu\,(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

✓ **For the onset of yielding :**

$$Y^2/2E=1/2E\,[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\,\nu\,(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

✓ **Yield function**

$$f = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \upsilon(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - Y^2$$

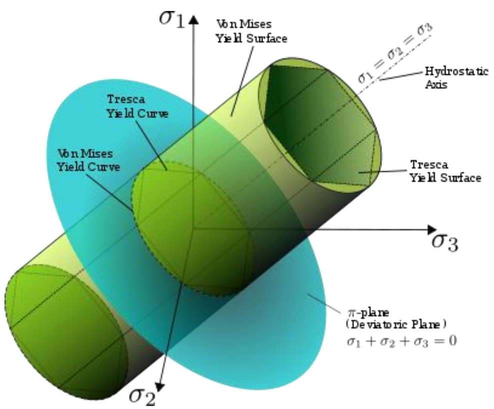
$$f = \sigma_e^2 - Y^2$$

$$\boxed{\text{Yielding} \Rightarrow f = 0,\ \ \text{safe } f < 0}$$

The axis of the cylinder passes through the origin of the coordinates for unyielded material

It is inclined equal amount to the three axes and represents pure hydrostatic stress for elastic deformations.

The effective stress is the uniaxial stress that is equally distant from the yield surface or located on it



The effective stress or the stress intensity for an elastic material is expressed as

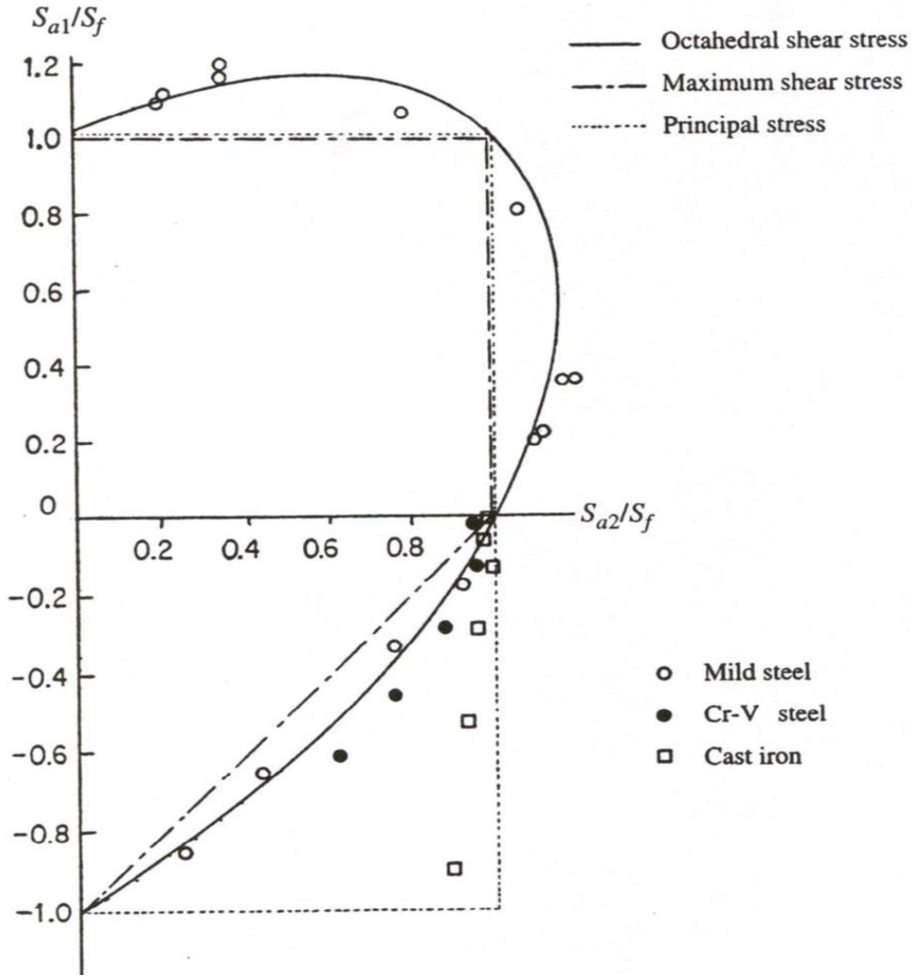
$$\sigma_{eff} = \frac{\sqrt{2}}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{yz}{}^2 + \tau_{zx}{}^2 + \tau_{xy}{}^2\right)}$$

And the effective strain as

$$\varepsilon_{eff} = \frac{\sqrt{2}}{2(1+\nu)}\sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \left(\varepsilon_y - \varepsilon_z\right)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}\left(\gamma_{yz}{}^2 + \gamma_{zx}{}^2 + \gamma_{xy}{}^2\right)}$$

And $\sigma_{eff} = E\varepsilon_{eff}$

Yield criteria for deformation of metals under plane stress



The data for the mild steel and Cr-V steel which behave in a ductile manner agree well with the octahedral shear stress (von Mises) criterion
Data for cast iron which behaves in a brittle manner, agrees better with the maximum principal stress criterion:

$$\sigma_1 = \sigma_y$$

Brittle materials fail suddenly in a tensile test by rupture without any prior yielding.

When a brittle material is under uniaxial tensile stress, the value of the normal stress which causes it to fail is equal to the ultimate strength of the material as determined from a tensile test.

When a brittle material is under plane stress, the principal stresses are compared to the ultimate strength obtained from the uniaxial tensile test.

Maximum principal stress criterion states that a brittle material will fail when the maximum normal stress exceeds the ultimate strength obtained from the uniaxial tensile test:

$$|\sigma_a| > \sigma_U \text{ or } \qquad |\sigma_b| > \sigma_U$$

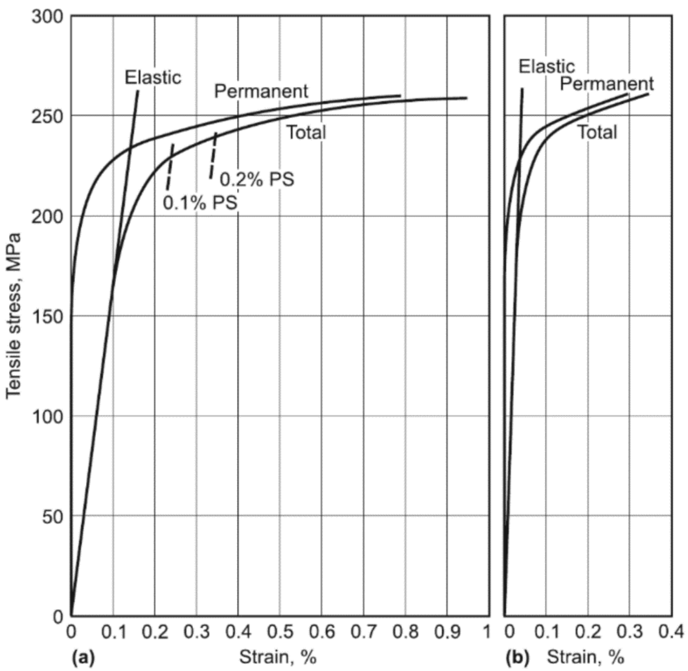
This criterion forms a square area centered on the xy plane. The criterion is based on the assumption that the ultimate strength of materials under tension and compression are equal, which is an overestimation for most materials as the presence of cracks and flaws often weaken the material under tension

Example – Evaluate the yielding stress condition for a ductile cast iron using maximum shear stress, maximum principal stress and maximum distortion energy criteria.

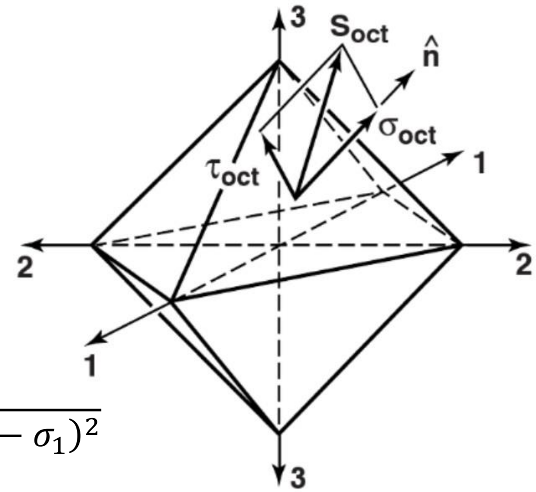
$|\sigma_a| > \sigma_Y$ or $|\sigma_b| > \sigma_Y$ or $(|\sigma_a| - |\sigma_b|) > \sigma_Y$

$|\sigma_a| > \sigma_U$ or $|\sigma_b| > \sigma_U$

$\sigma_a{}^2 - \sigma_a\sigma_b + \sigma_b{}^2 > \sigma_Y{}^2$



Prediction of yielding under multiaxial loading according to the maximum shear stress criterion involves the analysis of the octahedral planes



There are eight octahedral planes making equal angles with the principal stress directions

The shearing stress on these planes is given by

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Or with nonprinciple stresses:

$$\tau_{oct} = \frac{1}{3}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left({\tau_{yz}}^2 + {\tau_{zx}}^2 + {\tau_{xy}}^2\right)}$$

The shear strain acting on an octahedral plane is given by

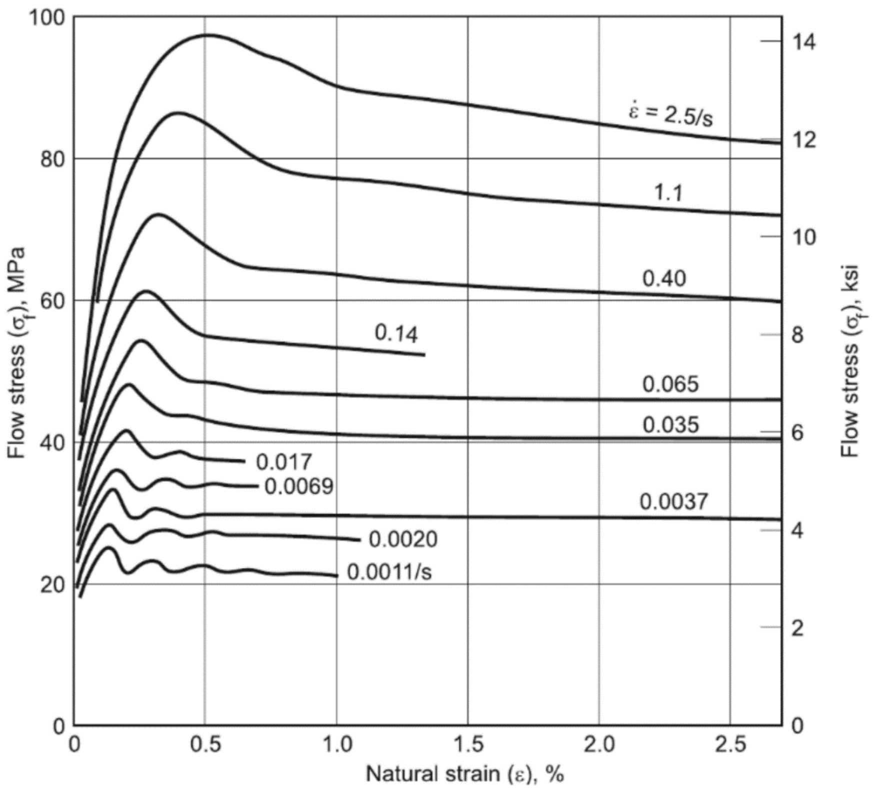
$$\gamma_{oct} = \frac{2}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$

Or

$$\gamma_{oct} = \frac{2}{3}\sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \left(\varepsilon_y - \varepsilon_z\right)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}\left({\gamma_{yz}}^2 + {\gamma_{zx}}^2 + {\gamma_{xy}}^2\right)}$$

Deformation and plastic behavior of metals

Effect of strain rate at high T

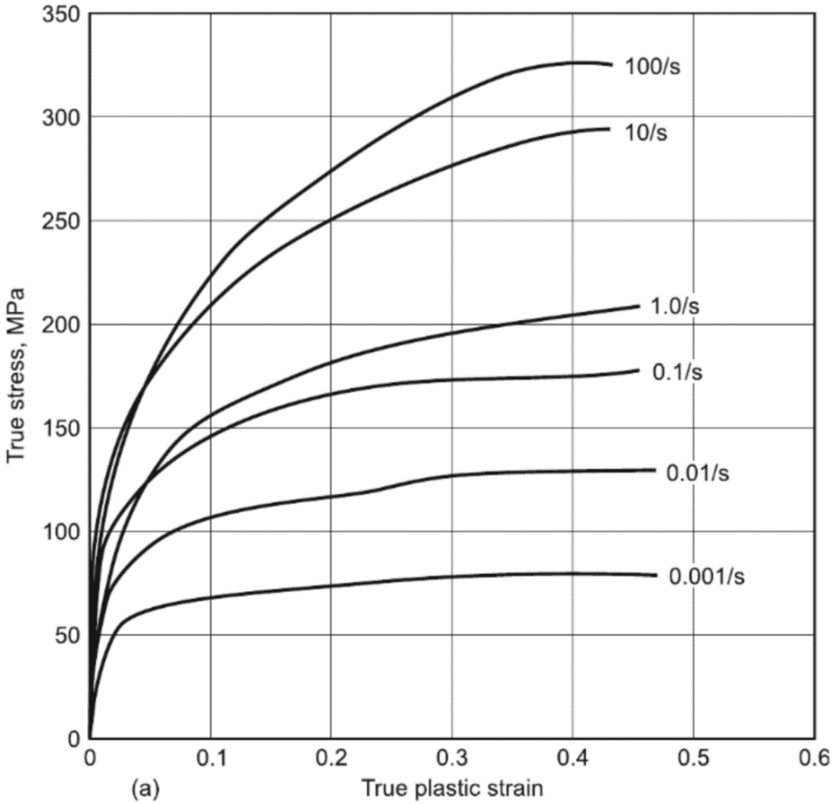


CS.032 1025 carbon (0.25% C) steel, flow stress-strain curves at various strain rates

Temperature (T) = 1100 °C (2012 °F). Stress-strain curves show that at higher strains the flow stress is approximately constant. This is increasingly true at smaller strain rates ($\dot{\varepsilon}$). Curves were obtained in hot torsion experiments. UNS G10250

Source: K. Lange, Ed., *Handbook of Metal Forming,* McGraw-Hill, 1985, p 16.11

Effect of strain rate at high T

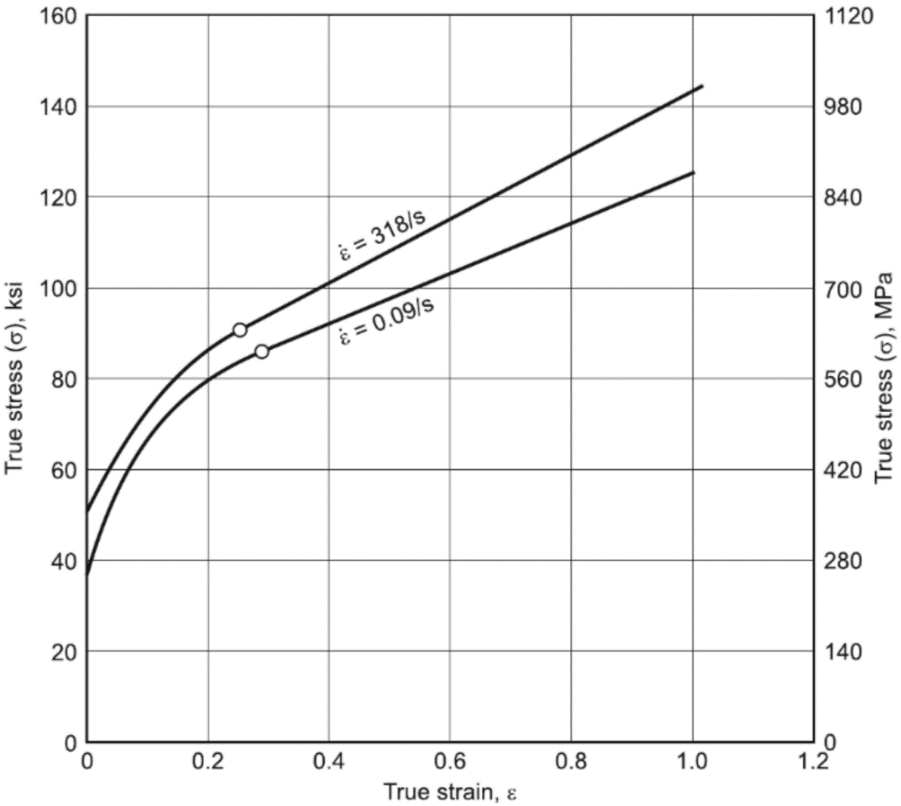


HS.005 Microalloyed high-strength low-alloy (HSLA) steel, compressive true stress-true plastic strain curves at different strain rates

Hot rolled. Thermomechanical processing typically includes rough rolling, 1100–1240 °C (2012–2264 °F), and finish rolling, 810–900 °C (1490–1652 °F), fast cooling to 700 °C (1292 °F), and air cooling. (a) Tested at 900 °C. (b) At 1200 °C. Composition: Fe-0.08C-1.3Mn-0.3Si-0.2Ni-0.08V-0.05Nb-0.015P-0.008S

Source: N.S. Mishra, in *Hot Working Guide A Compendium of Processing Maps*, Y.V.R.K Prasad and S. Sasidhara, Ed., ASM International, 1997, p 337

Effect of strain rate at low T

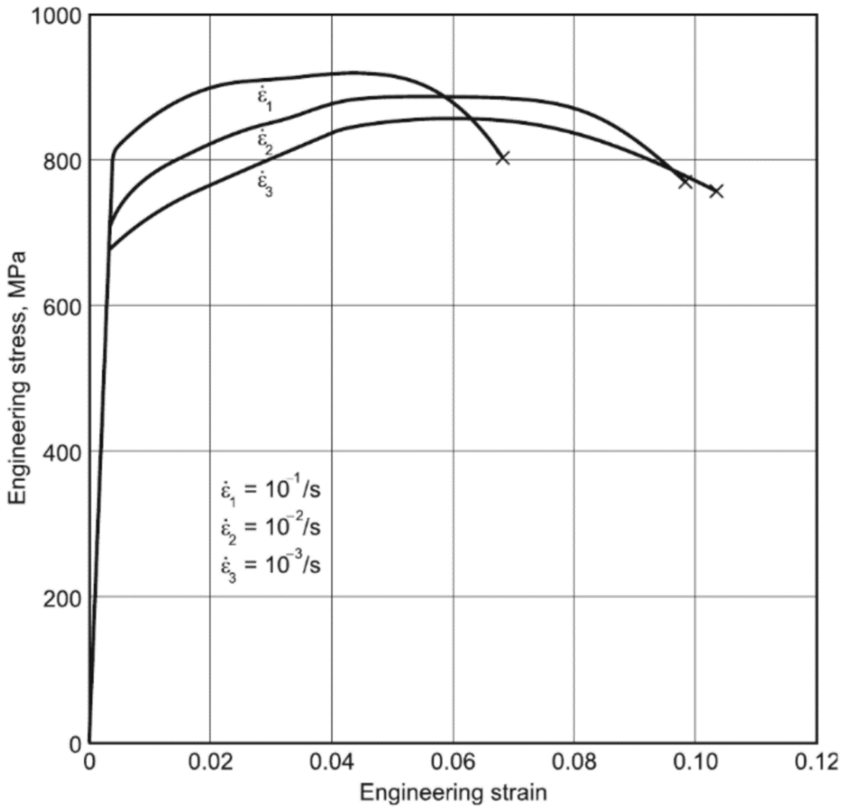


CS.038 1112 carbon steel, true stress-strain curves with effect of strain rate

True stress-strain curves for 1112 steel at different strain rates at 21 °C (70 °F). When metals are tested in tension at different strain rates, the flow stress corresponding to a given strain is found to increase with strain rate. The following equation is frequently used to relate flow stress and strain rate at a given strain and temperature: $\sigma = \sigma_1 \dot{\varepsilon}^m$, where $\dot{\varepsilon} = d\varepsilon/dt$ and σ_1 and m are material constants. The exponent m (strain-rate sensitivity) is found to increase with temperature, especially above the strain recrystallization temperature. In the hot-working region, metals tend to approach the behavior of a Newtonian liquid for which $m = 1$.

Source: M.C. Shaw, *Metal Cutting Principles*, Clarendon Press, Oxford, 1984, p 69

Effect of strain rate on yield strength

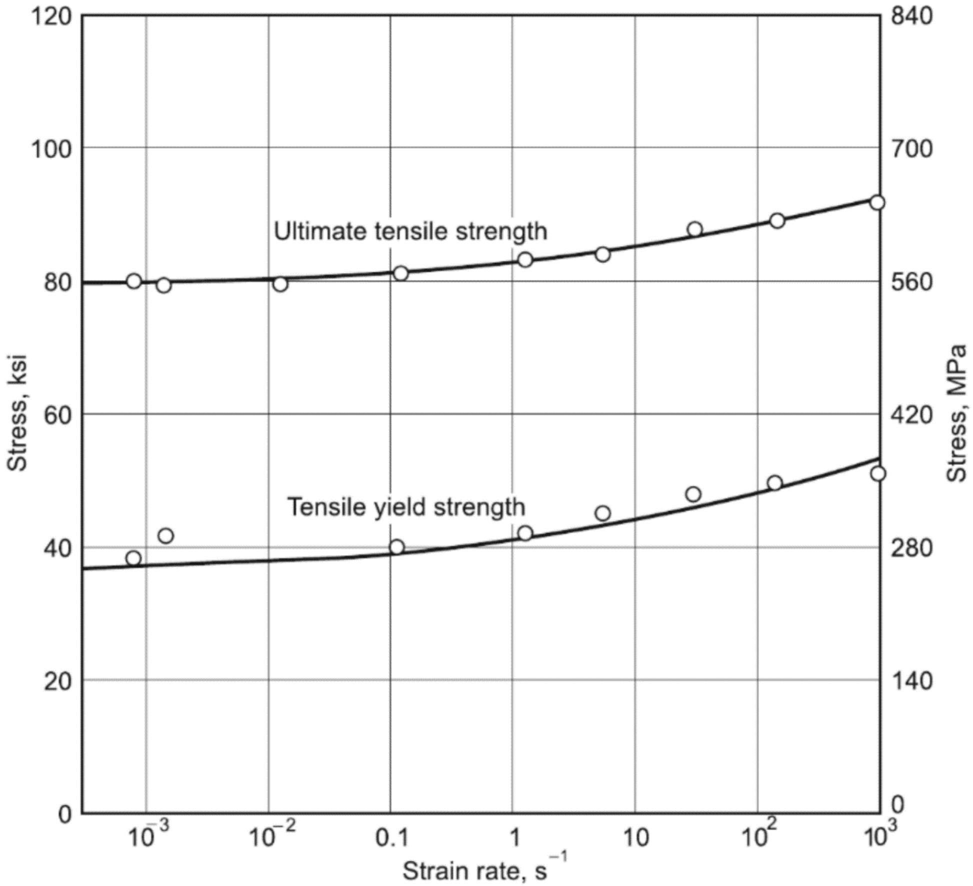


CS.033 1040 carbon steel, engineering stress-strain curves with effect of strain rate

Effect of different strain rates on the tensile response. The yield stress and flow stresses at different values of strain increase with strain rate. The work-hardening rate (m), on the other hand, is not as sensitive to strain rate. This illustrates the importance of correctly specifying the strain rate when giving the yield stress of a metal. Not all metals exhibit a high strain-rate sensitivity. Aluminum and some of its alloys have either 0 or $-m$. In general, m varies between 0.02 and 0.2 for homologous temperatures between 0 and 0.9 (90% of melting point in K). Therefore, one would have, at the most, an increase of 15% in the yield stress by doubling the strain rate. UNS G10400

Source: M.A. Meyers and K.K. Chawla, *Mechanical Metallurgy: Principles and Applications,* Prentice-Hall, 1984, p 572

Effect of strain rate on yield strength, UTS



SS.055 310 annealed stainless steel sheet, effect of strain rate on mechanical properties

Sheet thickness = 1.60 mm (0.063 in.). Composition: Fe-25Cr-20.5Ni. UNS S31000

Source: R.G. Davies and C.L. Magee, The Effect of Strain-Rate upon the Tensile Deformation of Metals, *J. Eng. Mater. Technol.*, April 1975, p 151. As published in *Aerospace Structural Metals Handbook,* Vol 2, Code 1305, CINDAS/USAF CRDA Handbooks Operation, Purdue University, 1995, p 22